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Fractal Geometry and Architecture

The built environment increasingly plays a determinant role in the lives of the citizens of this planet in the twenty-first century. With the earth's urban population recently exceeding 50% of all humans, places designed by architects and others dominate many aspects of daily life. Although an individual may become accustomed to such a constructed environment, this condition is not natural to our species. A study of evolution tells us the human animal evolved in the open savannah of the African continent, where nature created every aspect of the surroundings. All aspects of this environment contained fractal elements, creating a balance of order and complexity throughout the environment. Several hundred thousand years of conditioning our collective nature to this environment gives us innate responses to features that are pre-conscious, that is, we are affected by the environment in ways that we have no conscious control over. Designers rarely take into account these evolutionary factors in the design of our modern, urban habitat, and therefore create spaces, buildings and urban areas that can be alienating to their inhabitants. The lack of scale progression between essential elements of the modern urban fabric and within individual buildings is an entirely new condition for the human animal to exist within. By reviewing work from several disciplines, the current paper will argue that architects can positively affect pre-conscious reactions in building users through the use of considered scaling of built elements. Controlling scale progression may enable Architects to design structures that can elicit positive, pre-conscious reactions in building users.

It is useful to compare the current, modern urban environment in which we live with the natural one in which our species evolved. The African savannah is ingrained in human nature, being that it is where we became what we currently are over hundreds of thousands of years. Nature is a total and complete whole, where each element is in balance with its surroundings. The modern city, on the other hand, is not a complete whole made of discrete elements, but a collection of individual units grouped together for economic convenience. [figs. 1,2]

The savannah is composed of elements that demonstrate fractal ordering, from the distribution of sizes of animals relative to their population to the individual elements of a tree that offers refuge and views. Salingaros states "One can measure the population density of a particular type of animal and correlate it with its mass. The relationship between body mass and the relative abundance of different species in an ecosystem," has a direct fractal nature.[1] The tree that provided early humans with both refuge and prospect is also fractal in its individual elements. The trunk divides down into branches and then twigs and sticks that the leaves sprout from, themselves self-similar from the stem to the veins running out to the edges. [fig. 3]

There is no similar ordering method evident in the amalgamations of structures that define the modern city. Traditional cities that grew up before the industrial revolution and the material advance that it created display evidence of fractal ordering. Salingaros and West contend that "[t]he most successful urban regions all over the world are found to have a great range of connections, from footpaths, to bicycle paths, to low-traffic roads, to through roads, up to expressways: in decreasing number."[2] This distribution is similar to that of animals in an ecosystem, the smaller the path, the more of them there are, while the largest of these paths, the expressway, occur infrequently in the urban fabric. This condition is exceedingly rare in North American cities, where paths tend to be either medium sized or larger, and most buildings are either large commercial buildings or single family homes.

Neurological Considerations

We are wired to receive visual stimuli in specific ways. Our brain is just as active in creating a "reality" out of the stimuli we see as are our eyes when scanning the environment. To begin processing the vast amount of visual cues that are received by the visual regions of the brain from the eyes, through the optic nerve, the brain essentially searches for elements that are static or unchanging. Semir Zeki describes this phenomenon as "the law of constancy. This law is rooted in the fact that the brain is only interested in the constant, essential and non-changing properties of," the visual world "when the information reaching it is never constant from moment to moment."[3] One can infer from this that perhaps an architecture that incorporates scaled self-similar elements will pre-consciously please the visual brain.

Furthermore, research is beginning to suggest that we humans not only judge beauty and aesthetics subjectively but that there is also an objective component to what pleases us visually. In a groundbreaking study on brain responses to images of classical and renaissance sculpture, a team of Italian neuroscientists discovered that certain specific brain areas "light up", or become activated, when asked to perform discrete tasks. Cinzia Di Dio and her colleagues set up an experiment to observe neurological responses to passive and active tasks and categorize the results. Their use of canonical classic sculptures and renaissance imitations of the same is important for our purposes in the study of fractal form and proportion. These pieces are mostly formed around the ideal conception of the perfect human form, and all hold to the golden ratio, 1:1.618, as an ordering principle. This ratio is present in many naturally occurring fractal phenomena, like the spiral of a conch shell, and the distance between ears of corn on a stalk, among others. Her experiment took images of thirty of these sculptures and manipulated them by altering the proportions, either by elongating or shortening the torso or legs [fig. 4]. The images were then presented to her study subjects as unaltered originals. By asking her subjects to either simply observe the images, or to decide whether they were aesthetically or proportionally pleasing, she was able to make conclusions about the way the brain makes such judgments. The tests were conducted while the subjects were in an fMRI machine, and the set of images included 15 unaltered, canonical images and 15 images where the proportion of the sculpture had been changed. She concludes "we propose that the positive emotional *feeling* elicited in the viewer by the canonical images was determined by a preferential coding of these images, relative to the modified ones, by various cortical areas and by a concurrent, *joint* activation of the anterior insula."[4] (Di Dio's emphasis) This is in contrast to the images that were "judged" as either beautiful or ugly, which activated not the insula but the amygdala, which has a "fundamental…function…to provide neutral stimuli with positive or negative values through associative learning."[5]

Following these two observations, Di Dio makes this rather startling claim: "The main question we addressed in the present study was whether there is an objective beauty… Our results gave a positive answer to this question."[6] By determining that there is a biological and physiological positive response to images containing the golden ratio, this research has shown that this proportion can be employed by designers to elicit that beneficial pre-conscious response, and suggests that fractal elements in design will do the same.

Fractals

Our species, homo sapiens, has been around in roughly its current form for hundreds of thousands of years. During the vast majority of that time, we were a nomadic hunter/gatherer animal, living in groups of individuals, searching for sustenance and protection in a world that was as untamed as ourselves. It is estimated that organized agriculture began around 6 millennia ago, bringing with it the earliest forms of "civilization". Looking at this fact, one can conclude that, whatever the impetus for creating settlements may have been, our species began down a path away from the environment in which we evolved and towards one that we create for ourselves. Periodic advancements in technology brought us the industrial revolution and the invention of the modern city. As stated previously, over half of the human population now lives in urban environments. This is a condition to which our animal self is not adapted, and while it is impractical and undesirable to say that we all go back to living as our pre-civilized ancestors may have, we can reintroduce elements of the natural world into the one we build for ourselves. One basic quality that pervaded our evolutionary environment was the fractal nature of the surroundings, providing a balance of complexity and order. It is the purpose of the current paper to define what fractals are and propose methods of introducing fractal qualities into architecture.

For our purposes, we will use Carl Bovill's definition of fractal geometry as... "the formal study of self-similar structures,"[7]. In a complex system, a repeating pattern is present, whether it is readily apparent to the human eye or it requires analysis, at different scales. We have discussed this above concerning the distribution of population size of animals in an ecosystem to their relative body mass, and also found a pattern in the number of paths in an urban area relative to their traffic intensity and size. These are but two examples of the multiplicity rule, a rather complex mathematical concept that is expounded upon by Salingaros.[8] Other methods of fractal generation and analysis exist and should be studied to give us, as designers, a broad range of tools and techniques for producing architecture that incorporates self-similar geometries. First we will give an overview of geometric fractals and their generation. Next we will dive into the techniques of analysis and generation of fractal forms to search for potential applications for architects.

The Cantor set, named after the nineteenth century German mathematician, is a relatively simple fractal to generate [fig. 5]. It is derived by removing the central ⅓ of a line segment. Once this operation is completed, it is repeated, and on and on. The result is fractal, but mainly confined to a linear condition.

Another fractal generation, Koch's snowflake [fig. 6], introduces the second dimension. Whereas the Cantor set removes the middle third of a line at each step of the process, the Koch curve instead inserts two segments of the same length which create two legs of an equilateral triangle. By repeating this process on each new line segment, down to infinity, a line segment of infinite length is theoretically created even though it has ends.

Bovill describes the Box Counting method as a tool to determine the fractal "dimension" of geometric compositions. This technique allows for the comparison of fractal qualities between different subjects. The method begins by placing a square grid over the subject in question and counting the number of squares that the pattern being studied occupies. The grid is then reduced in size and the number of squares again is calculated. Placing the first number in relation to the second will result in a figure. A perfect fractal, like Koch's snowflake, will always generate the same result regardless of the size of the boxes in the grid, whereas natural features, like coastlines, or manmade geometries, such as building elevations, will generally have a range over which they demonstrate their fractal nature. (Bovill, 73)[8]. Bovill subjected the work of two famous and prolific twentieth century architects to this box counting method, with interesting results. Frank Lloyd Wright's Robie House [fig. 7] in Chicago was analyzed both in elevation and at the individual window level. This analysis shows that the long elevation has a fractal dimension between 1.6 and 1.4 as the boxes drop in size from 24 feet to 3 feet, and analysis of a typical casement window from the house resulted in a range of 1.7 to 1.6 with boxes from 6 inches down to 1½". Bovill suggests "[t]his is not surprising since [Wright's] writings about design keep coming back to observation of the deep structure of nature."[9] Villa Savoye, the domestic warning shot of modernism designed by Le Corbusier [fig. 8], reveals an unsurprising contrast. The elevation of the building has a fractal dimension of 1.4 when the boxes are 8 feet per side, but all fractal nature disappears as the boxes decrease in size to 1 foot. (Bovill,73)[10] This very well may be intentional, as Le Corbusier was championing a turn towards a new architecture and away from traditional materials and methods of construction. Modernism had no place for nature, even at an intrinsic geometric level.

Bovill also performs fractal analysis on less prominent architecture. By employing Range Analysis and the Hurst Dimension, derived from studying the time intervals between floods on rivers, Bovill proposes that vernacular architecture displays a multiple number of fractal scales, "The results showed that there was more than one ordering of the architectural elements that produced a consistent fractal calculation across a range of scales and that different orderings produced different fractal measurements."[11] One can conclude from this analysis that individual craftsmen working independently, with consistent materials and methods, cannot but help create works that refer to each other in very specific, fractal ways. The next conclusion would be that this fact lies in the inherent qualities of the materials, which in vernacular architecture are invariably natural, local materials, known to the craftsmen with techniques that grew out of the region.

Another form of fractal generation that architects may employ is curdling, a process the produces a random pattern of “fractal dust”. The random nature of the results may lead to its employ only in aesthetic purposes rather than using it as an organizing principle for structure or laying out a plan. To produce a “fractal dust” pattern through curdling, one begins with a standard grid and randomly chooses a set of squares. This process is repeated until a satisfactory result is obtained. We can employ this method to perhaps perforate a screen wall or to apply a pattern to what would otherwise be an unadorned expanse of built structure. The copper facade of the recent de Young Museum in San Francisco’s Golden Gate Park [fig. 9], although its pattern is image based, begins to demonstrate this idea.

An informative study was conducted by the researcher Daniele Capo. She focuses on the three “architectural orders” as defined by Andrea Palladio in his treatise The Four Books of Architecture. This is a very good place to begin studying fractals in architecture because these works arguably form the basis for the last several hundred years of our trade. Analysis is of the Doric, Corinthian, and the Composite orders of column, or the engaged column, and a control virtual column composed through application of the Canter set. She uses a modified version of the box counting method to determine the fractal coefficient of each order and the control column. In the “information dimension” method of fractal analysis, the number of points in each square, not just the number of squares occupied, becomes the determining factor. With this method Capo “shows that all three of Palladio’s orders maintain a certain consistency of the data up to the eighth level, indicating that the value of the dimension is demolished only when the count is based on squares with a side that is equal to 1/256th of the height of the entire order.”[11] If one were to “consider an order height of 10 meters, then fractal coherence is maintained down to a detail of 4 centimeters.”[12] When we consider the pedigree of the Palladian orders, it should not be that surprising that fractal qualities are so deeply ingrained. Palladio was quantifying the architecture of the Romans, from centuries before him. The Romans were merely copying how the ancient Greeks built temples out of stone. And the stone temples that the Greeks built were merely a more permanent interpretation of structures made from wood and reeds that came before. The fractal nature of the Palladian orders appears to be a direct descendant of the ancient Greek method of construction, which had no other source of inspiration than what was available in nature, and that, of course, was itself entirely fractal.

As an example of a contemporary building that uses the idea of fractal geometry in its design, we will look Federation Square in Melbourne [figs. 11, 12]. This complex, designed by lab architecture, uses principles of fractal geometry in the design of the curtain wall facade. By employing a common triangular rain screen panel shape, which can differ in material and orientation, the architects attempt to visually link the facades of nine buildings in a single complex. Some of the buildings require opaque facades while others want to remain transparent. The triangular panels also begin to articulate the façade in the third dimension, creating openings for entries, windows and skylights. By employing a single repeating shape over the entire project, lab architecture creates a cohesive whole out of several discrete buildings.

In conclusion, the contemporary urban environment is one that is quite different than our species' evolutionary landscape. Solid materials, rectilinear geometries and abrupt changes in scale are all commonplace to you and I, and most people on the planet at this moment in time. However, this is a recent condition. We became the animals that we are in nature, an environment that had fractal characteristics everywhere one looked. While these fractal relationships may not have been apparent to our ancestors, the fact they were omnipresent became ingrained in our psyche.

We, as humans, are accustomed to being in an environment rich in fractal characteristics. The modern city has, for the first time in human existence, created an environment devoid of such fractal geometries. By incorporating the study and toil of several diverse fields of study, one can begin applying fractal ideas and geometries into the projects and buildings that architects design and create environments that respond to and nurture this essential human nature.

1. Salingaros, N.A. and West, B.J., 1999 "A Universal Rule for the Distribution of Sizes", Environment and Planning B: Planning and Design 26, 909-923.
2. Salingaros, N.A. and West, B.J., 1999 "A Universal Rule for the Distribution of Sizes", Environment and Planning B: Planning and Design 26, 909-923.
3. Zeki, S., 2004 "The Neurology of Ambiguity", Consciousness and Cognition 13, 173-196.
4. Di Dio, C., Macaluso, E., Rizzolatti, G., 2007 "The Golden Beauty: Brain Responses to Classical and Renaissance Sculptures", PLoS ONE 11, 1-8
5. Di Dio, C., Macaluso, E., Rizzolatti, G., 2007 "The Golden Beauty: Brain Responses to Classical and Renaissance Sculptures", PLoS ONE 11, 1-8
6. Di Dio, C., Macaluso, E., Rizzolatti, G., 2007 "The Golden Beauty: Brain Responses to Classical and Renaissance Sculptures", PLoS ONE 11, 1-8
7. Bovill, C., 2000 "Fractal Geometry as Design Aid", Journal for Geometry and Graphics 4 71-78.
8. Salingaros, N.A. and West, B.J., 1999 "A Universal Rule for the Distribution of Sizes", Environment and Planning B: Planning and Design 26, 909-923.
9. Bovill, C., 2000 "Fractal Geometry as Design Aid", Journal for Geometry and Graphics 4 71-78.
10. Bovill, C., 2000 "Fractal Geometry as Design Aid", Journal for Geometry and Graphics 4 71-78.
11. Capo, D., 2004 "The Fractal Nature of the Architectural Orders", Nexus Network Journal 6 30-41.
12. Capo, D., 2004 "The Fractal Nature of the Architectural Orders", Nexus Network Journal 6 30-41.

Fractals and Architecture, images addendum

figure 1: Human's evolutionary environment, an African savannah

figure 2: Human's contemporary environment, a manufactured city

figure 3: elm trees displaying fractal characteristics

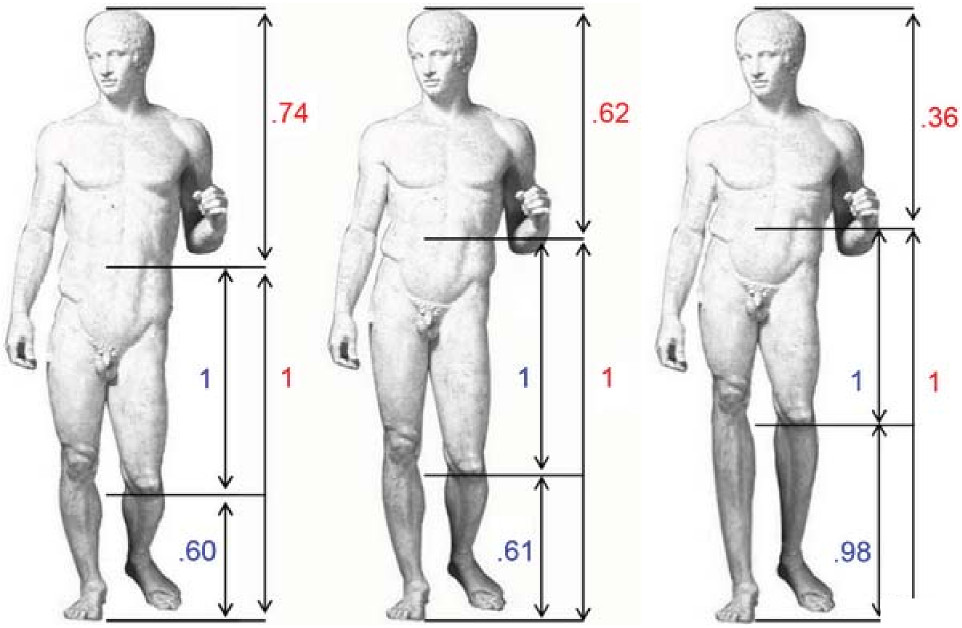

figure 4: an example of Di Dio's study images

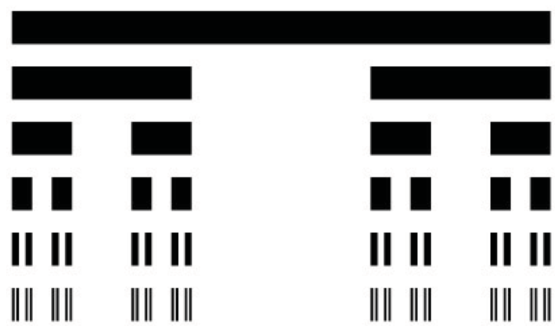

figure 5: the Cantor Set

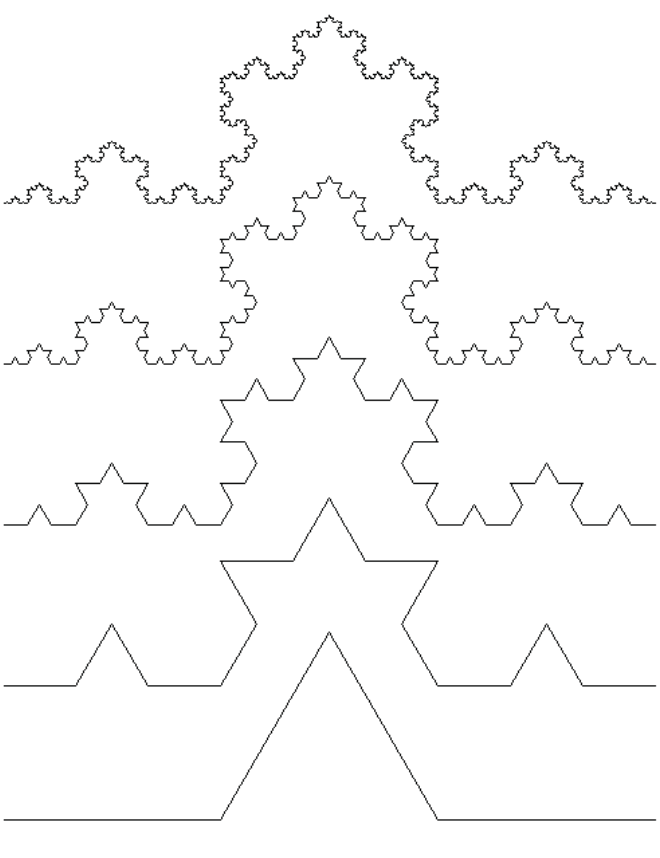

figure 6: Koch's Snowflake

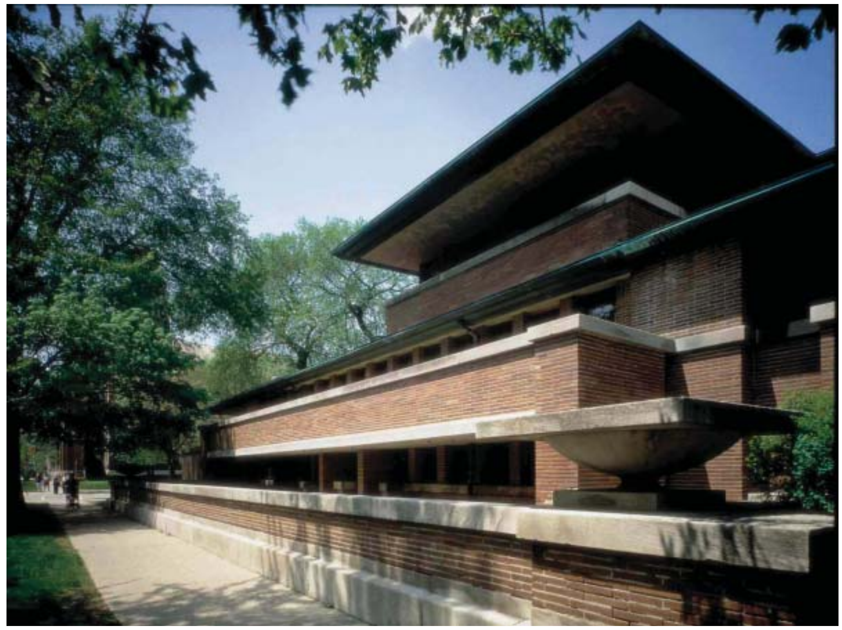
figure 7: Wright's Robie House


figure 8: Le Corbusier's Villa Savoye

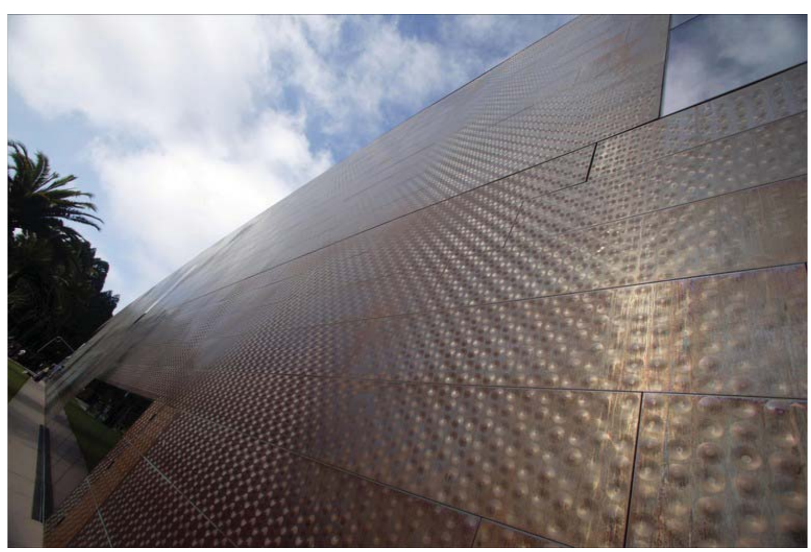
figure 9: the articulated facade of the de Young Museum

Fractals and Architecture, images addendum

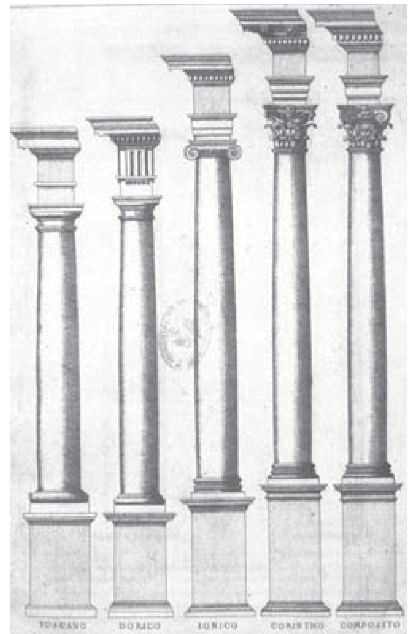

figure 10: full height orders from Palladio's Four Books on Architecture

figures 11 and 12: lab architecture's Federation Square in Melbourne. Small triangles form bigger triangles forming facade systems